Phys 402
Fall 2022
Homework 1
Due Wednesday, 7 September @ 10 AM as a PDF upload to ELMS

1. Find $\langle r \rangle$ and $\langle r^2 \rangle$ for an electron in the ground state of the hydrogen atom. {*Hint: neither of these expectation values are zero!*}

2. Starting with the time-independent Schrödinger equation in spherical coordinates, Griffiths 3rd Edition Eq. (4.14), separate variables and derive the "θ equation" for $y(x) = \Theta(\theta)$ by making the substitution $x = \cos\theta$. The result is the Associated Legendre equation:

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + \left[\ell(\ell+1) - \frac{m^2}{1-x^2}\right]y = 0,$$

where the separation constants were written as $\ell(\ell+1)$ and m^2, as in Griffiths. By taking $m = 0$ (giving the Legendre differential equation), and using a series solution method around $x = 0$, show that ℓ must be an integer to keep the solutions finite near the regular singular points $x = \pm 1$.

3. Starting with radial part of the Schrödinger equation (Griffiths (4.35)), with $V(r)$ for the Hydrogen atom, make the substitution $u(r) = rR(r)$ and obtain (4.53). Solve this equation by the power series method after "stripping off" the asymptotic behavior for $u(r)$ in the limits of small and large r (see section 4.2.1). From the resulting recursion relation for the expansion coefficients examine the nature of the solutions at large r. Conclude what must be done to keep the wave function finite at large r, and determine the resulting eigenenergy spectrum. Find the (un-normalized) ground state radial wavefunction $R(r)$.

4. Griffiths and Schroeter *Quantum Mechanics*, 3rd Ed., Problem 4.29 (Pauli spin matrices and their commutation relations)

5. An entangled state is one that cannot be written as a product of single-particle states. Take the $|10\rangle$ state of two spin-1/2 particles as an example. Try to write it as a product of the most general single particle spinors of the two particles: $|\psi_1\rangle = \alpha_1|\uparrow\rangle + \beta_1|\downarrow\rangle$ for spin number 1 and $|\psi_2\rangle = \alpha_2|\uparrow\rangle + \beta_2|\downarrow\rangle$ for spin number 2. Compare this to the entangled state $|1\quad 0\rangle = \frac{1}{\sqrt{2}}\left(|\downarrow\rangle|\uparrow\rangle + |\uparrow\rangle|\downarrow\rangle\right)$. What do you conclude? In an entangled state the state of particle 1 depends on the state of particle 2, and *vice versa*.

6. Griffiths and Schroeter *Quantum Mechanics*, 3rd Ed., Problem 4.64 (Superposition state of the hydrogen atom and total angular momentum $\vec{J} = \vec{L} + \vec{S}$. *Hint: for parts (e) and (f) make use of the Clebsch-Gordan coefficients to convert the wavefunction from the L,S-basis to the J-basis*.)

Continued on the next page …

EXTRA CREDIT

1. Griffiths and Schroeter *Quantum Mechanics*, 3rd Ed., Problem 4.59 (Dot product of spins in a singlet state)